AD662358

הטכניון - מכון טכנולוגי לישראל

הפקולטה להנדסה אוירונוטית

TECHNION – ISRAEL INSTITUTE OF TECHNOLOGY

DEPARTMENT OF AERONAUTICAL ENGINEERING

HAIFA, ISRAEL

62

AF 61(052)-905

AUGUST 1967

SR - 2

SCIENTIFIC REPORT No. 2.

BUCKLING OF DISCRETELY RING STIFFENED CYLINDRICAL SHELLS.

Josef Singer and Raphael Haftka

Technion - Israel Institute of Technology
Department of Aeronautical Engineering,
Haifa, Israel.

TAE REPORT No. 67.

The research reported in this document has been sponsored by the AIR FORCE OFFICE OF SCIENTIFIC RESEARCH OAR under Contract AF 61(052) - 905 through the European Office of Aerospace Research, United States Air Force.

SUMMARY

The buckling of ring-stiffened cylinders is studied by a "discrete" approach, in which the rings are considered as linear discontinuities represented by the Dirac delta function. The analysis is a linear Donnell type theory that takes account of the eccentricity of stiffeners. Buckling loads under hydrostatic pressure, lateral pressure and axial compression are compared with those obtained by "smeared-stiffener" theory for an extensive range of geometries. The discreteness effect depends very strongly on the geometry of the shell and the eccentricity of the rings. Significant discreteness effects are found for hydrostatic pressure loading.

LIST OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SYMBOLS

A_n	coefficient of axial displacement
A_{1j}	cross sectional area of j^{th} stringer
A_{2i}	cross sectional area of i^{th} ring
a	distance between rings
B_n	coefficient of circumferential displacement
b	distance between stringers
b_{m1}, b_{m2}	expressions defined by Eqs. (9)
C_n	coefficient of radial displacement
c_{m1}, c_{m2}	expressions defined by Eqs. (9)
D	$[Eh^3/12(1-\nu^2)]$
E, E_1, E_2	moduli of elasticity of shell, stringers and rings respectively
e_1, e_2	distance between centroid of stiffener cross section and middle surface of shell, positive when stiffener is inside
G_1, G_2	shear moduli of stringers and rings respectively
h	thickness of shell
I_{11}, I_{22}	moment of inertia of stiffener cross section about its centroidal axis
I_{o1}, I_{o2}	moment of inertia of stiffener cross section about the middle surface of the shell
I_{t1}, I_{t2}	torsion constant of stiffener cross section
L	length of shell between bulkheads

M_x, M_ϕ, $M_{x\phi}$	moment resultants acting on element
N_x, N_ϕ, $N_{x\phi}$	membrane force resultants acting on element
N_{xo}, $N_{\phi o}$, $N_{x\phi o}$	prebuckling membrane force resultants
n	number of stringers
P	axial load
p	lateral or hydrostatic pressure
q	number of subshells for ring stiffened cylinder (number of rings is q-1)
R	radius of shell
t	number of circumferential waves
u	non-dimensional axial displacement $(=u^*/R)$
u^*	axial displacement
v	non-dimensional circumferential displacement $(=v^*/R)$
v^*	circumferential displacement
w	non-dimensional radial displacement $(=w^*/R)$
w^*	radial displacement
x	non-dimensional axial coordinate $(=x^*/R)$
x^*	axial coordinate
Z	$(1-\nu^2)^{1/2}(L/R)^2(R/h)$

β	$\pi R/L$
β_{1i}, β_{2i}	expressions defined by Eqs. (9)
δ_{imn}, γ_{1i}, γ_{2i}	expressions defined by Eqs. (9)
ζ_{1j}	$E_{1j}A_{1j}e_{1j}/D$
ζ_{2i}	$E_{2i}A_{2i}e_{2i}/D$
η_{o1j}	$E_{1j}I_{o1j}/RD$
η_{o2i}	$E_{2i}I_{o2i}/RD$
η_{t1j}	$G_{1j}I_{t1j}/RD$
η_{t2i}	$G_{2i}I_{t2i}/RD$
λ	$(PR/\pi D)$
λ_p	$(R^3/D)p$
μ_{1j}	$(1-\nu^2)(E_{1j}A_{1j}/ERh)$
μ_{2i}	$(1-\nu^2)(E_{2i}A_{2i}/ERh)$
ν	Poisson's ratio
ϕ	circumferential coordinate
χ_{1j}	$[(1-\nu^2)E_{1j}A_{1j}e_{1j}/EhR^2]$
χ_{2i}	$[(1-\nu^2)(E_{2i}A_{2i}e_{2i}/EhR^2)]$

Subscripts following a comma indicate differentiation

Subscripts i and j refer to i'th ring and j'th stringer respectively.

I. INTRODUCTION

Until a few years ago the customary approach to the stability analysis of a stiffened shell was to replace it by an equivalent orthotropic shell (see for example [1] - [3]). This approach, however, did not permit taking into account the eccentricity of stiffeners which was found to be of importance in heavily stiffened shells (for example [4] or [5]). A second approach also assumes the stiffeners to be "smeared" over the whole surface of the shell, but considers the "distributed" stiffness of the stiffeners separately, which permits inclusion of the effect of eccentricity. The eccentricity effect had been pointed out long ago, [6] and [7], but was studied in detail only in recent years (see for example [8], [9], [10], [11], [12] or [13]).

The "smearing" of stiffeners appears reasonable for shells with many closely spaced stiffeners, but is doubtful when their number is small. The study of discretely stiffened shells is therefore of interest.

Hence the buckling of stiffened cylindrical shells is analysed by a third approach in which, instead of "smearing" the stiffeners, they are considered as linear discontinuities represented by the Dirac delta function. This representation is satisfactory as long as the width of the stiffeners is not comparable to the distance between them. One may add that a "discrete" analysis includes also consideration of local instability between stiffeners, whereas the first two approaches dealt with general instability only.

This approach has been employed in [14] for buckling of cylindrical shells under torsion, in [15] for buckling under lateral pressure, and in [16] and [17] for the case of axial compression. In these analyses, however, the eccentricity of stiffeners was not considered. In [18] the equilibrium and stability equations for discretely stiffened shells are derived taking into account the eccentricity of stiffeners. The present study uses the formulation of [18] and some preliminary results for ring-stiffened cylindrical shells were given in [19].

It should be pointed out that the effect of discreteness of rings on the buckling of cylindrical shells under lateral pressure has been investigated in [20] by an analog method, and that parallel work on axially compressed shells by the approach employed here has recently been reported in [21]. These studies, however, treat only typical examples and do not extend to a wide range of geometries.

In this report ring-stiffened cylindrical shells under lateral or hydrostatic pressure and axial compression are studied in detail. The main purpose is to find the difference between the buckling loads predicted by discrete and "smeared" theories and the influence of the various geometrical parameters on this difference.

The rings considered are equally spaced, but not necessarily of constant cross-section. For the cases of varying ring cross-section, the variation is assumed symmetrical with respect to the mid-length of the shell, since there is no physical motivation for asymetrical stiffness distribution.

The analysis is a linear Donnell type theory and for clarity the main assumptions are repeated here:

(1) The shell is thin, $(R/h) \gg 1$, and hence high powers of (h/R) are neglected.

(2) The number of circumferential waves is not small, or rather $t^2 \gg 1$.

(3) The rings are concentrated in their p. es, i.e. they have zero width in the axial direction.

(4) The normal strains $\varepsilon_x(z)$ and $\varepsilon_\phi(z)$ vary linearly in the ring as well as in the sheet. The normal strains in the ring and in the sheet are equal at their point of contact.

(5) The rings do not transmit shear out to their plane, hence the shear membrane force $N_{x\phi}$ is carried entirely by the sheet.

(6) The rings carry torsional moments on account of their torsional rigidity.

2. STABILITY EQUATIONS

The stability equations and force and moment expressions for a cylindrical shell stiffened by rings and stringers derived in [18] are given here for convenience (it should be remembered that in [18] the stringers are assumed to be equal, whereas every ring may have a different cross section):

$$0 = \delta U = \int_{o}^{L/R} \int_{o}^{2\pi} \{[- N_{x,x} - N_{x\phi,\phi}] R\delta u$$

$$+ [- N_{\phi,\phi} - N_{x\phi,x}] R\delta v$$

$$+ [- M_{x,xx} - RN_{\phi} - M_{\phi,\phi\phi} + M_{x\phi,x\phi} - M_{\phi x,x\phi}$$

$$- R (N_{xo} w_{,x})_{,x} - R (N_{\phi o} w_{,\phi})_{,\phi} - R (N_{x\phi o} w_{,\phi})_{,x}$$

$$- R (N_{x\phi o} w_{,x})_{,\phi}] \delta w \} Rdxd\phi$$

$$+ \int_{o}^{2\pi} [N_x R\delta u + N_{x\phi} R\delta v - M_x \delta(w_{,x})$$

$$+ (M_{x,x} - M_{x\phi,\phi} + M_{\phi x,\phi} + RN_{xo} w_{,x}$$

$$+ RN_{x\phi o} w_{,\phi}) \delta w]_{x=o}^{x=L/R} Rd\phi \tag{1}$$

and

$$N_x = \frac{Eh}{(1-\nu^2)} \{[u_{,x} + \nu(u_{,\phi} - w)] + \sum_{j=1}^{n} \delta(\phi - \frac{2\pi j}{n})(\mu_1 u_{j,x}$$

$$- \chi_1 w_{j,xx})\}$$

$$N_\phi = \frac{Eh}{(1-\nu^2)} (v_{,\phi} - w + \nu u_{,x}) + \sum_{i=1}^{q-1} \delta (x - \frac{i}{q} \frac{L}{R})[\mu_{2i}(v_{i,\phi} - w_i)$$

$$- \chi_{2i} w_{i,\phi\phi}]\}$$

$$N_{x\phi} = N_{\phi x} = \frac{Eh}{2(1+\nu)} (u_{,\phi} + v_{,x}) \tag{2}$$

$$M_x = \left(-\frac{D}{R}\right)\left[\left(w_{,xx} + \nu w_{,\phi\phi}\right) + \sum_{j=1}^{n} \delta\left(\phi - \frac{2\pi j}{n}\right)\left(\eta_{o1} w_{j,xx} - \zeta_1 u_{j,x}\right)\right]$$

$$M_\phi = \left(-\frac{D}{R}\right)\left\{\left(w_{,\phi\phi} + \nu w_{,xx}\right) + \sum_{i=1}^{q-1} \delta\left(x - \frac{iL}{qR}\right)\left[\eta_{o2i}\, w_{i,\phi\phi}\right.\right.$$

$$\left.\left. - \zeta_{2i}\left(v_{i,\phi} - w_i\right)\right]\right\}$$

$$M_{x\phi} = \left(\frac{D}{R}\right)\left[(1-\nu)w_{,x\phi} + \sum_{j=1}^{n} \delta\left(\phi \quad \frac{j}{n} 2\pi\right)\eta_{t1} w_{j,x\phi}\right]$$

$$M_{\phi x} = \left(-\frac{D}{R}\right)\left[(1-\nu)w_{,x\phi} + \sum_{i=1}^{q-1} \delta\left(x - \frac{iL}{qR}\right)\eta_{t2i} w_{i,x\phi}\right] \qquad (3)$$

After substitution of Eqs. (2) and (3) into Eq. (1) and omission of the terms that relate to stringers, the stability equation for a ring-stiffened cylindrical shell becomes:

$$0 = \delta U = \int_0^{L/R} \int_0^{2\pi} \left\{ - \frac{Eh}{(1-\nu^2)} \left\{u_{,xx} + \frac{1-\nu}{2} u_{,\phi\phi} + \frac{1+\nu}{2} v_{,x\phi} - \nu w_{,x}\right\} R\delta u\right.$$

$$- \frac{Eh}{(1-\nu^2)} \left\{\left(\frac{1+\nu}{2} u_{,x\phi} + v_{,\phi\phi} + \frac{1-\nu}{2} v_{,xx} - w_{,\phi}\right)\right.$$

$$\left. + \sum_{i=1}^{q-1} \delta\left(x - \frac{iL}{qR}\right)\left[\mu_{2i}\left(v_{i,\phi\phi} - w_{i,\phi}\right) - \chi_{2i} w_{i,\phi\phi\phi}\right]\right\} R\delta v$$

$$+ \frac{D}{R} \left\{\left[w_{,xxxx} + 2w_{,xx\phi\phi} + w_{,\phi\phi\phi\phi} + 12\left(\frac{R}{h}\right)^2 \left(w - v_{,\phi} - \nu u_{,x}\right)\right]\right.$$

$$+ \sum_{i=1}^{q-1} \delta\left(x - \frac{i}{q} \frac{L}{R}\right)\left[\zeta_{2i}\left(2w_{i,\phi\phi} - v_{i,\phi\phi\phi}\right) + \eta_{o2i} w_{i,\phi\phi\phi\phi}\right.$$

$$\left. + 12 \left(\frac{R}{h}\right)^2 \mu_{2i}\left(w_i - v_{i,\phi}\right)\right] + \sum_{i=1}^{q-1} \delta_{,x}\left(x - \frac{iL}{qR}\right) \eta_{t2i}\, w_{i,x\phi\phi}$$

$$+ \frac{R^2}{D}[(N_{xo}w_{,x})_{,x} + (N_{\phi o}w_{,\phi})_{,\phi} + (N_{x\phi o}w_{,x})_{,\phi} + (N_{x\phi o}w_{,\phi})_{,x}]\}\delta w\Big\} R\, dx\, d\phi$$

$$+ \int_0^{2\pi}\Big\{\frac{Eh}{(1-\nu^2)}\{[u_{,x} + \nu(v_{,\phi} - w)]\} R\delta u$$

$$+ \frac{Eh}{2(1+\nu)}(u_{,\phi} + v_{,x})R\delta v + \frac{D}{R}\{w_{,xx} + \nu w_{,\phi\phi}\}\delta(w_{,x})$$

$$- \frac{D}{R}\{[w_{,xxx} + (2-\nu)w_{,x\phi\phi}] - \frac{R^2}{D}(N_{xo}w_{,x}$$

$$+ N_{x\phi o}w_{,\phi})\}\delta w\Big\}_{x=o}^{x=L/R} R d\phi \qquad (4)$$

3. SOLUTION

The displacement components are expanded into Fourier series in the axial direction

$$u = \sin t\phi \sum_{n=1}^{\infty} A_n \cos n\beta x$$

$$v = \cos t\phi \sum_{n=1}^{\infty} B_n \sin n\beta x \qquad (5)$$

$$w = \sin t\phi \sum_{n=1}^{\infty} C_n \sin n\beta x$$

Each set of terms of series (5) satisfies the stability equations of a shell with smeared stiffeners (Eqs. 12 of [8]) and each term fulfils the classical simple support boundary conditions

$$\left.\begin{aligned} w &= 0 \\ M_x &= 0 \\ v &= 0 \\ N_x &= 0 \end{aligned}\right\} \quad \text{at } x = 0,\ (L/R) \qquad (6)$$

Substitution of the displacements (5) into Eq. (4) yields:

$$0 = \int_0^{L/R}\int_0^{2\pi}\Bigg\{-\frac{Eh}{(1-\nu^2)}\sum_{n=1}\{-n^2\beta^2A_n - (\frac{1-\nu}{2})t^2A_n$$

$$- (\frac{1+\nu}{2})tn\beta\, B_n - \nu n\beta C_n\}\cos n\beta x \sin t\phi \sum_{m=1}^{\infty} R\delta A_m \cos m\beta x \sin t\phi$$

$$-\frac{Eh}{(1-\nu^2)}\sum_{n=1}^{\infty}\{-(\frac{1+\nu}{2})tn\beta A_n - t^2B_n - (\frac{1-\nu}{2})n^2\beta^2B_n$$

$$- tC_n + \sum_{i=1}^{q-1}\delta(x - \frac{iL}{qR})[-\mu_{2i}t^2B_n - \mu_{2i}tC_n$$

$$+\chi_{2i}t^3C_n]\}\, R \sin n\beta x \cos t\phi \sum_{m=1}^{\infty} \sin m\beta x \cos t\phi\, \delta B_m$$

$$+\frac{D}{R}\Bigg\{\sum_{n=1}^{\infty} C_n\,\{(n^2\beta^2 + t^2)^2 + 12\,(\frac{R}{h})^2\,[C_n + tB_n + \nu n\beta\, A_n]$$

$$+\sum_{i=1}^{q-1}\delta(x - \frac{iL}{qR})\,[\zeta_{2i}(-2t^2C_n - t^3B_n)$$

$$+\eta_{o2i}\, t^4C_n + 12(\frac{R}{h})^2\,\mu_{2i}(C_n + tB_n)]$$

$$+ \frac{R^2}{D} [C_n(- n^2\beta^2 N_{xo} - t^2 N_{\phi o} + 2tn\beta N_{x\phi o}]\} \sin n\beta x \sin t\phi$$

$$+ \sum_{i=1}^{q-1} \delta_{,x} (x - \frac{iL}{qR}) n\beta t^2 \, n_{+2i} \, C_n \sin t\phi \cos n\beta x \Bigg\} \sum_{m=1}^{\infty} \delta C_m \sin m\beta x \sin t\phi \Bigg\} R \, dx \, d\phi \qquad (7)$$

It was tacticly assumed in Eq. (7) that the prebuckling stresses N_{xo}, $N_{\phi o}$ and $N_{x\phi o}$ are constant all over the cylinder. In the case of hydrostatic or lateral pressure and axial compression this assumption is valid, if the membrane stresses represent the prebuckling stresses satisfactorily. Since each of the coefficients of δA_m, δB_m and δC_m (m = 1 .... ∞) must be zero, Eq. (7) is equivalent to three infinite sets of algebraic equations.

The first set (that corresponding to δA_n) does not contain delta functions and can be solved directly by equating the integrand to zero.

$$- A_n [n^2\beta^2 + (\frac{1-\nu}{2})t^2] - B_n (\frac{1+\nu}{2}) tn\beta - C_n \nu n\beta = 0$$

or

$$A_n = \frac{-\nu n\beta C_n + tn\beta(\frac{1+\nu}{2})B_n}{n^2\beta^2 + (\frac{1-\nu}{2}) t^2} \qquad (8)$$

Substitution of Eq. (8) into the non-vanishing parts of Eq. (7) yields after appropriate integrations two sets of equations

$$b_{m1}\, B_m + c_{m1}\, C_m + \sum_{n=1}^{\infty} \sum_{i=1}^{q-1} (\beta_{11} B_n + \gamma_{11} C_n) \sin \frac{mi\pi}{q} \sin \frac{ni\pi}{q} = 0$$

$$b_{m2}\, B_m + c_{m2}\, C_m + \sum_{n=1}^{\infty} \sum_{i=1}^{q-1} (\beta_{12} B_n + \gamma_{12} C_n) \sin \frac{mi\pi}{q} \sin \frac{ni\pi}{q} + \delta_{imn} C_n \cos \frac{mi\pi}{q} \cos \frac{ni\pi}{q} = 0$$

$$m = 1 \ldots\ldots \infty \qquad (9)$$

where

$$b_{m1} = \left[\frac{(\frac{1+\nu}{2})^2 t^2 m^2 \beta^2}{m^2\beta^2 + (\frac{1-\nu}{2}) t^2} - t^2 - (\frac{1-\nu}{2}) m^2 \beta^2 \right] \frac{L}{2R}$$

$$c_{m1} = \left[\frac{\nu (\frac{1+\nu}{2}) m^2 \beta^2 t}{m^2\beta^2 + (\frac{1-\nu}{2})\, t^2} - t \right] \frac{L}{2R}$$

$$b_{m2} = - 12 \left(\frac{R}{h}\right)^2 c_{m1}$$

$$c_{m2} = \left\{ (m^2\beta^2 + t^2)^2 + 12 \left(\frac{R}{h}\right)^2 \left[1 - \frac{\nu^2 m^2 \beta^2}{m^2\beta^2 + (\frac{1-\nu}{2})\, t^2} \right] + \frac{R^2}{D} (-n^2\beta^2 N_{xo} - t^2 N_{\phi o} + 2tn\beta N_{x\phi o}) \right\} \frac{L}{2R}$$

$$\beta_{11} = -\mu_{2i}$$

$$\gamma_{11} = \chi_{2i} t^3 - \mu_{2i} t$$

$$\beta_{12} = -\zeta_{2i} t^3 + 12 \left(\frac{R}{h}\right)^2 \mu_{2i} t$$

$$\gamma_{21} = -2\zeta_{2i} t^2 + \eta_{o2i} t^4 + 12 \left(\frac{R}{h}\right)^2 \mu_{2i}$$

$$\delta_{imn} = mn\beta^2 t^2\, \eta_{t2i}$$

4. THE STABILITY DETERMINANT.

The buckling load is found from the vanishing of the determinant of Eqs. (9).

For n terms (or n equations) the determinant is of the order 2n. As shown in Appendix I, the equations separate into subgroups. Hence the stability determinant is reduced into subdeterminants. By choosing one equation (not necessarily the first one) one obtains a first approximation to the buckling load. It is also shown in Appendix II that in the case of uniform rings this approximation coincides with the "smeared" solution. In this case, the difference between the results of "discrete" theory and "smeared" theory is therefore the difference between the first order approximation and that of infinite order.

5. LOCAL BUCKLING

Obviously the effect of discreteness of rings is more pronounced for a small number of rings than for many rings. When there are only few rings, however, local buckling between rings usually occurs at a lower load than general instability.

When the torsional stiffness of the rings is neglected, the local buckling load is that of a simply supported cylinder of the same thickness and radius but of length L/q. Or, in other words, the local buckling load is equivalent to that of an unstiffened cylinder forced to buckle with $n = q$.

In Table I buckling loads are given for several wave numbers n. Comparison of this table with Table I of reference [11] (the complete report, TAE 42) shows that, in most cases of fairly strong rings, more than 3 rings are needed for general instability to be predominant (to yield the lower buckling loads). In some cases more than 20 rings are needed.

Hence the effect of discreteness of rings is studied in particular for shells with weak rings.

6. NUMERICAL RESULTS AND DISCUSSION

The critical pressure was computed for discretely stiffened cylinders covering a wide range of geometries. Most of the calculations relate to cylinders stiffened with one ring only, since the discreteness effect is most pronounced for one ring. Furthermore the results for one ring are more accurate, as discussed in Appendix III, in connection with convergence considerations.

As the local buckling load is very low for a cylinder stiffened with one ring, very weak rings are considered to ensure that the general instability loads will be lower.

6.1. Hydrostatic Pressure

Figure 2 shows the influence of the shell parameters L/R and R/h on the effect of discreteness. The effect is presented as the difference between the "discrete" and "smeared" loads (or rather the percentage reduction in

buckling load for "discrete" rings) versus the Batdorf shell geometry parameter $Z = (1-\nu^2)^{1/2}(R/h)(L/R)^2$, that combines L/R and R/h in the appropriate manner. Shells with one ring are considered for which the difference is most pronounced. It is seen that the dependence is monotonous and there is very small scatter. The scatter is mainly caused by the necessary integer values of the number of circumferential waves, that produce ripples in the separate "smeared" and "discrete" curves, the difference of which is presented in Fig. 2.

Rings with the following geometric parameters $e_2/h = 1$, $A_2/ah = 0.1$, $I_{22}/ah^3 = 0.01$ were chosen since they ensure for all shell geometries general instability at about 60%-80% of the local buckling load. This is high enough (see also Fig. 3) for a pronounced discreteness effect, but on the other hand not too close to the cut-off line (that represents the local buckling load).

Figure 3 shows the influence of the relative stiffness of the rings on the discreteness effect. When the rings are very weak, the buckling mode tends to remain in simple sinusoidal form, as for unstiffened shells, and the "discreteness"effect is therefore very small. For stronger rings, the shell seeks lower energy modes and hence the difference between the "smeared" and "discrete" theory becomes more pronounced. Again shells with one ring are considered, for which the ring spacing a = L/2. In the example of Fig. 2, the ring eccentricity is 1 and hence, for rings of rectangular cross-section, (A_2/ah) would be (c/a) = (2c/L) where c is the width of the ring. For rings of rectangular cross-section (I_{22}/ah^3) would then be 0.0833(c/a) = 0.0833(2c/L)

In Fig. 3 the ring cross-section differs slightly from the rectangular and is defined by $(A_2/ah) = (2c/L)$ and $(I_{22}/ah^3) = 0.1(2c/L)$. $f = (2c/L)$ appears as a parameter in the curves of Fig. 2.

Figure 3 also shows the effect is larger for external rings than for internal ones. The general behavior is however similar.

Figure 4 shows that effect of the eccentricity on the discreteness effect. Comparison of rings of equal A_2/ah and I_{22}/ah^3 would not be appropriate here, since the eccentricity noticeably affects the "smeared" buckling load, and the comparison would then actually be of rings of different strengths instead. The comparison in Fig. 4 is between rings of equal strength with cross-sections defined by $A_2/ah = 0.1f$, $I_{22}/ah^3 = 0.01f$ and where the parameter f was adjusted for each eccentricity to yield the same range of "smeared" buckling load.

As was pointed earlier, Figs. 2 to 4 show the influence of the discreteness effect for shells with one ring only. Figs. 5 and 6 show dependence of the effect on the number of rings. The general trend is, as expected, for the magnitude of the difference to go asymptotically to zero with increasing number of rings. It is however seen that for high L/R and R/h, when the one-ring difference is highest, the effect also decreases more slowly with increasing number of rings.

It should be noted that the results here are the 10^{th} approximation and are not accurate for one or two rings for high L/R.

In Tables 2 to 6 some of the numerical results on which Figs. 2 to 7 are based are presented in tabular form. The values in the tables are in some cases given up to 5 or six significant figures, as obtained from the computer program, though usually only the first 4 figures are meaningful.

Figure 6 summarizes the importance of the discreteness of rings in the instability analysis of cylindrical shells under hydrostatic pressure. It shows the variation of the percentage reduction in buckling pressure (due to discreteness of the rings) with number of rings and increasing ring strength. As the "smeared" stiffness - represented in the figure by the critical pressure parameter of the stiffened shell - increases, the discreteness effect for I ring rises till the cutoff point is reached, when local buckling becomes dominant for I ring. For 2 rings the discreteness effect is at first smaller, but then rises beyond that for I ring, till the cutoff point for 2 rings is reached. This increase in discreteness effect with number of rings and ring strength continues till at 6 rings a maximum for this geometric configuration is reached. Then the effect starts to decrease with increase in number and strength of rings.

It should be noted that the largest discreteness effect does not occur with one ring, as one would intuitively assume, since local buckling becomes dominant before the ring can attain a high stiffness that would result in an appreciable discreteness effect, There are two opposing tendencies, one for the discreteness effect to decrease with number of rings for a given basic shell, and the other a postponement of the cut off point with number of rings. The

result is a certain maximum for every shell geometry as shown in Fig. 6.

6.2. Lateral Pressure with Non-Uniform Rings.

In order to investigate the effect of discreteness of stiffeners for non-uniform stiffeners, the buckling loads were computed for ring-stiffened cylinders under lateral pressure with rings of varying cross-section. Results of a "smeared" stiffener theory are available for sinusoidal height variation of rings with rectangular cross-section [22]. For the "smeared" case, the height of the rings d as a function of x can be written

$$d = d_o + d_1 \sin\beta x \tag{10}$$

where d_o represents a constant part and d_1 the amplitude of the varying portion of the ring cross-section. For the corresponding discrete case the height of rings is d_i $i = 1 \ldots (q-1)$ and the i^{th} ring is at $x_i = \frac{iL}{qR}$. The discretely varying portion of the height of the ring, d_i is chosen so that the mean height is the same as for the "smeared" case and also that

$$\frac{d_i - d_o}{d_j - d_o} = \frac{\sin x_i}{\sin x_j} \tag{11}$$

In Table 7 the critical lateral pressures are given for three non-uniformly stiffened shells with 3 rings. The pressure parameters for the non-uniform and uniform discrete cases are compared with the corresponding "smeared" theory values from [22]. It is seen that the difference between the uniform and non-uniform cases is considerable in the discrete analysis. This does not

invalidate the weight savings predicted by the "smeared" theory in [22]. For, even in the 3-ring case, the gain in critical pressure predicted by the "smeared" theory can be recovered, if a larger portion of the height is varied than in [22]. The results, however, indicate that for shells with few rings the optimal dimensions predicted by "smeared" theory may have to be revised in view of the discreteness effects. Discrete theory is therefore needed for a reliable analysis when one wishes to exploit the advantages of non-uniform stiffening for shells with few rings.

6.3. Axial Compression

The critical load of unstiffened cylinders under axial compression is a function of the wave parameter

$$\frac{(n^2\beta^2 + t^2)^2}{n^2\beta^2}$$

and not of t and n independently. (This applies strictly only to "classical" simple supports and to shells that buckle into many waves in the axial direction). Hence in many cases a ring-stiffened cylindrical shell under axial compression can buckle under the same load as a corresponding unstiffened cylinder, since for $n=q$ it buckles through the rings for some value of the wave parameter. This "ineffectiveness" of rings disappears only for large β or large n - for shells with small ring spacing. These shells, however, exhibit a very small discreteness effect (see shells Nos. 4 to 8 in Table 8). As small ring spacing means low effective Z, this is not surprising if one remembers that also for the case of

hydrostatic pressure only very small discreteness effects were found for shells of small Z.

In Table 8, eight typical ring-stiffened shells, chosen from the test data of [23] and [24], are studied for discreteness effects. Shells Nos. 1 to 3 show a very small, but noticeable discreteness effects. The local buckling load in these shells, for simple supports, would be slightly below the general instability, which would mean that the results presented are beyond the cut-off point and therefore meaningless; but as in reality there is some restraint at the boundaries of the sub-shells, the cut-off point is shifted and the results are of value. For shells Nos. 4 to 8, the local buckling load is larger than the general instability load, even for simple supports. The discreteness effect is, however, negligible as already pointed out above.

The computations of Table 8 neglect the rotational restraint of the rings (n_{t2} = 0 is assumed). Calculations with the proper n_{t2} values are in progress. It can, however, be expected that the conclusion - that the discreteness effect is very small for general instability of ring-stiffened cylinders under axial compression - will not be affected.

7. CONCLUSIONS

The results of the calculations for cylindrical shells under hydrostatic or lateral pressure show that the discreteness effect depends very strongly on the geometry of the shell and the eccentricity of the rings.

Though for most practical cases with many rings the effect is not important, one should be cautious when dealing with shells of high Z and rings of low eccentricity. The "discrete" load may then be about 20% lower than the "smeared" one even for ten rings or more.

In ring-stiffened shells under axial compression the discreteness effect is always very small.

REFERENCES

1) Bodner S.R., General Instability of a Ring-Stiffened Circular Cylindrical Shell under Hydrostatic Pressure, Journal of Applied Mechanics Vol. 24, No. 2, 269-277, June 1957.

2) Becker, H. and Gerard, G., Elastic Stability of Orthotropic Shells. Journal of the Aerospace Sciences, Vol.29, No.5, 505-512, May 1962.

3) Singer, J. and Fersht-Scher, R., Buckling of Orthotropic Conical Shells Under External Pressure:, The Aeronautical Quarterly, Vol. 15, Part 2, 151-168, May 1964.

4) Card, M.F., Preliminary Results of Compression Tests on Cylinders with Eccentric Longitudinal Stiffeners, NASA TM X -1004, September 1964.

5) De Luzio, A.J., Stuhlman, C.E. and Almroth, B.O., Influence of Stiffener Eccentricity and End Moment on the Stability of Cylinders in Compression, Presented at the AIAA 6th Structures and Materials Conference, Palm Springs, 5-7 April 1965.

6) Flügge, W., Die Stabilität der Kreiszylinderschale, Ingenieur Archiv, Vol.3, 463-506, 1932.

7) Van der Neut, A., The General Instability of Stiffened Cylindrical Shells Under Axial Compression, Report S-314, National Luchtvaartlaboratorium, Amsterdam, Report and Transactions, Vol. 13, S.57-84, 1947.

8) Baruch, M. and Singer, J., The Effect of Eccentricity on the General Instability of Stiffened Cylindrical Shells Under Hydrostatic Pressure, Journal of Mechanical Engineering Science, Vol. 5, No. 1, 23-27, March 1963.

9) Hedgepeth, J.M. and Hall, D.B., Stability of Stiffened Cylinders, AIAA Journal Vol. 3, No. 12, 2275-2286, December 1965.

10) Block, D.L., Card, M.F. and Mikulas, M.M., Buckling of Eccentrically Stiffened Orhotropic Cylinders, NASA TN D-2960, August 1965.

11) Singer, J., Baruch, M. and Harari, O., Inversion of the Eccentricity Effect in Stiffened Cylindrical Shells Buckling under External Pressure, Journal of Mechanical Engineering Science, Vol. 8, No. 4, 363-373, December 1966. Also TAE REport 42, Technion Research and Development Foundation, Haifa, Israel, August 1965.

12) Singer, J., Baruch, M. and Harari, O., On the Stability of Eccentrically Stiffened Cylindrical Shells under Axial Compression, TAE Report 44, Technion Research and Development Foundation, Haifa, Israel, December 1965 (to be published in the International Journal of Solids and Structures).

13) Seggelke, P. and Geier, B., Das Beulverhalteh versteifter Zylinderschalen, Teil 2, Beullasten. (to be published in Zeitschrift für Flugwissenschaften).

14) Stein, M., Sanders, S.L. and Crate, H., Critical Stress of Ring-Stiffened Cylinders, NACA Report 989, 1951.

15) Moe, J., Stability of Rib-Reinforced Cylindrical Shells under Lateral Pressure, Publications Inter. Ass. Bridge Struc. Eng., Vol. 18, 113-136, 1958.

16) Block, D.L., Influence of Ring Stiffeners on Instability of Orthotropic Cylinders in Axial Compression, NASA TN D 2482, October 1964.

17) Van der Neut, A., General Instability of Orthogonally Stiffened Cylindrical Shells, Collected Papers on Instability of Shell Structures - 1962, NASA TN D-1510, 1962, pp. 309-319.

18) Baruch, M., Equilibrium and Stability Equations for Discretely Stiffened Shells, Israel Journal of Technology, Vol. 3, No. 2, 138-146, June 1965.

19) Singer, J. and Baruch M., Recent Studies on Optimization for Elastic Stability of Cylindrical and Conical Shells, presented at the Royal Aeronautical Society Centenary Congress, London, September 12-16, 1966.

20) MacNeal, R.H., Winemiller, A.F. and Bailie J.A., Elastic Stability of Cylindrical Shells Reinforced by One or Two Frames and Subjected to External Radial Pressure, AIAA Journal, Vol. 4, No.8, 1431-1433, August 1966.

21) Block D.L., Influence of Prebuckling Deformations, Ring Stiffeners and Load Eccentricity on the Buckling of Stiffened Cylinders, presented at the AIAA/ASME 8th Structures Structural Dynamics and Materials Conference, Palm Springs, California, March 29-31, 1967.

22) Harari, O., Singer, J. and Baruch, M., General Instability of Cylindrical Shells with Non-Uniform Stiffeners, Proceedings of 9th Israel Annual Conference on Aviation and Astronautics, Israel Journal of Technology, Vol. 5, No.1, 114-128, February 1967.

33. Almroth, B.O., Influence of Imperfections and Edge Restraint on the Buckling of Axially Compressed Cylinders" presented at the AIAA/ASME 7th Structures and Materials Conference, Cocoa Beach, Florida, April 18-20, 1966.

24. Milligan R., Gerard, G., Lakshmikantham, C., and Becker, H., " General Instability of Orthotropic Stiffened Cylinders under Axial Compression" Report AFFDL-TR-65-161, Air Force Flight Dynamics Laboratory, USAF, Wright Patterson Air Force Base, Ohio, July 1965.

ACKNOWLEDGEMENT

The authors would like to thank Miss A. Adler for assistance with the computations and the staff of the Technion Computing Center for their valuable help.

APPENDIX I.

REDUCTION IN ORDER AND SIMPLIFICATION OF STABILITY DETERMINANTS.

Equations (9) consist of two infinite sets of equations. The n^{th} approximation yields 2n equations and the corresponding stability determinant consists of n^2 blocks of 2 x 2. Denoting the blocks Δij we have

$$\Delta mm = \begin{vmatrix} b_{m1} + \sum_{i=1}^{q-1} \beta_{i1} \sin^2 \frac{m\pi i}{q} & c_{m1} + \sum_{i=1}^{q-1} \gamma_{i1} \sin^2 \frac{m\pi i}{q} \\ b_{m2} + \sum_{i=1}^{q-1} \beta_{i2} \sin^2 \frac{m\pi i}{q} & c_{m2} + \sum_{i=1}^{q-1} \gamma_{i2} \sin^2 \frac{m\pi i}{q} + \delta_{imm} \cos^2 \frac{m\pi i}{q} \end{vmatrix}$$

and ... I - I

$$\Delta mn = \begin{vmatrix} \sum_{i=1}^{q-1} \beta_{i1} \sin \frac{m\pi i}{q} \sin \frac{n\pi i}{q} & \sum_{i=1}^{q-1} \gamma_{i1} \sin \frac{m\pi i}{q} \sin \frac{n\pi i}{q} \\ \sum_{i=1}^{q-1} \beta_{i2} \sin \frac{m\pi i}{q} \sin \frac{n\pi i}{q} & \sum_{i=1}^{q-1} \gamma_{i2} \sin \frac{m\pi i}{q} \sin \frac{n\pi i}{q} + \delta_{imn} \cos \frac{m\pi i}{q} \cos \frac{n\pi i}{q} \end{vmatrix}$$

$m \neq n$

... I - 2

It is assumed that the rings are symmetrical with respect to the mid-length of the cylinders. Hence $\beta_{i1} = \beta_{(q-i)1}$; $\gamma_{i1} = \gamma_{(q-i)1}$ etc.

A sum of the form $\sum_{i=1}^{q-1} \alpha_i \sin \frac{m\pi i}{q} \sin \frac{n\pi i}{q}$ or $\sum_{i=1}^{q-1} \phi_i \cos \frac{m\pi i}{q} \cos \frac{n\pi i}{q}$ is zero when m is odd and n is even, or vice versa. This can be shown as follows:

$$\alpha_{(q-i)} \sin \frac{m\pi(q-i)}{q} \sin \frac{n\pi(q-i)}{q}$$

$$= \quad \alpha_i \sin \left(m\pi - \frac{m\pi i}{q}\right) \sin \left(n\pi - \frac{n\pi i}{q}\right)$$

$$= - \quad \alpha_i \sin \frac{m\pi i}{q} \sin \frac{n\pi i}{q} \qquad \ldots\ 1 - 3$$

since for odd m

$$\sin \left(m\pi - \frac{m\pi i}{q}\right) = \sin \frac{m\pi i}{q}$$

and for even n

$$\sin \left(n\pi - \frac{n\pi i}{q}\right) = - \sin \frac{n\pi i}{q}$$

The i^{th} term in the sum therefore cancels the $(q-i)^{th}$ term. Hence all the blocks Δmn for which m is odd and n is even, or m is even and n is odd, vanish. The stability determinant reduces therefore to two subdeterminants, the first containing the odd n's and second the even n's.

This reduction of the stability determinant applies to the most general case. In some particular cases there are further possible subdivisions.

The first case is that of rings with zero torsional stiffeness.

Rings of Zero Torsional Stiffness

Here $\delta_{imn} = 0$ and there are no sums of the type $\sum_{i=1}^{q-1} \phi_i \cos\frac{m\pi i}{q} \cos\frac{n\pi i}{q}$

The remaining sums, of the type $\sum_{i=1}^{q-1} \alpha_i \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q}$ are zero when m or n are equal to a multiple of q. This means that the equations numbered q, 2q, 3q... separate and the corresponding buckling load can be calculated separately.

The physical reason is immediately apparent. These particular mode of buckling are the modes of local buckling, and when the rings have zero torsional stiffness local buckling is the same as buckling of the unstiffened shell (a problem that has a closed form solution). Other subdivisions are possible when all the rings are equal.

Equal Rings of Zero Torsional Stiffness

When all the rings are equal the sums of the type $\sum_{i=1}^{q-1} \alpha_i \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q}$ become $\alpha \sum_{i=1}^{q-1} \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q}$. The sum

$$S_{mnq} = \sum_{i=1}^{q-1} \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q} \qquad \ldots \text{I} - 4$$

vanishes if neither m + n or m - n are multiples of 2q or if both are.

This may be shown as follows:

$$\sum_{i=1}^{q-1} \sin\frac{mi\pi}{q} \sin\frac{ni\pi}{q} = \frac{1}{2} \sum_{i=1}^{q-1} \left[\cos\frac{(m-n)i\pi}{q} - \cos\frac{(m+n)i\pi}{q} \right] \qquad \ldots \text{I} - 5$$

Now if $(m - n)$ is a multiple of $2q$

$$\sum_{i=1}^{q-1} \cos \frac{(m-n)i\pi}{q} = q - 1 \qquad \ldots 1 - 6$$

and the first set of terms of the sum equals $\frac{q-1}{2}$.

If $(m + n)$ is a multiple of $2q$

$$\sum_{i=1}^{q-1} \cos \frac{(m+n)i\pi}{q} = q - 1 \qquad \ldots 1 - 7$$

and the second set of terms of the sum equals $- \frac{q-1}{2}$. Hence when both $(m-n)$ and $(m + n)$ are multiples of $2q$,

$$S_{mnq} = 0 \qquad \ldots 1 - 8$$

To consider the case when neither $m + n$ nor $m - n$ are multiples of $2q$, denote $(m-n) = r$ and $(m+n) = s$. Then

$$\sum_{i=1}^{q-1} \cos \frac{(m-n)i\pi}{q} = \sum_{i=1}^{q-1} \cos \frac{ri\pi}{q} = \frac{1}{2} \sum_{i=1}^{q-1} (e^{\frac{jri\pi}{q}} + e^{-\frac{jri\pi}{q}}) \qquad \ldots 1 - 9$$

where j denotes here $j = \sqrt{-1}$.

Since all the blocks Δmn for which m is odd and n is even, or m is even and n is odd, vanish in the general case, only even values of r and s need be considered here.

Now

$$\sum_{i=1}^{q-1} e^{\frac{jri\pi}{q}} = \frac{e^{\frac{jr\pi}{q}}\left(1 - e^{\frac{jr(q-1)\pi}{q}}\right)}{1 - e^{\frac{jr\pi}{q}}} = -1 \quad \text{for even } r \qquad \ldots 1 - 10$$

Similarly

$$\sum_{i=1}^{q-1} e^{-\frac{jri\pi}{q}} = -1 \quad \text{for even } r \qquad \ldots 1 - 11$$

Hence, here,

$$\sum_{i=1}^{q-1} \cos \frac{(m-n)i\pi}{q} = -1 \qquad \ldots 1 - 12$$

In the same manner

$$\sum_{i=1}^{q-1} \cos \frac{(m+n)i\pi}{q} = \sum_{i=1}^{q-1} \cos \frac{si\pi}{q} = -1 \quad \text{for even } s \qquad \ldots 1 - 13$$

Therefore

$$S_{mnq} = \frac{1}{2} \sum_{i=1}^{q-1} \left[\cos \frac{(m-n)i\pi}{q} - \cos \frac{(m+n)i\pi}{q} \right] = 0 \qquad \ldots 1 - 14$$

and the subdeterminants are further simplified.

APPENDIX II

COMPARISON OF "SMEARED" AND "DISCRETE" SOLUTIONS.

To compare the "discrete" and "smeared" solutions one first has to clarify what are the corresponding cases.

The equivalence is defined here by taking the ring spacing as $a = \frac{L}{q}$.

The ring parameters μ_2, χ_2, ζ_2, η_{o2} as defined for the "smeared" case contain $\frac{1}{a}$. In the "discrete" theory $\frac{1}{a}$ is replaced in the parameter by $\frac{1}{R}$. Hence

$$\mu_{2s} = \frac{\mu_{2d} \cdot qR}{L} \qquad \ldots \text{II - 1}$$

where the subscript s refers to parameters as defined in the "smeared" theory and the subscript d to those defined in the "discrete" theory.

The "smeared" solution is in a closed form whereas in the "discrete" theory the solution is the limit of a series.

It can be shown that the first term, the first approximation, of the "discrete" solution is equal to the "smeared" solution, for the case of equal rings. If one compares Eq. (12) of Ref. [3] with Eq. (4) of this report, and solves these equations by the Galerkin method (choosing u, v, w as in Eq.(5)) one sees that the terms corresponding to shell itself are equal.

For the rings, there are in the "smeared" case terms of the type

$$S(m,n) = \mu_{2s} \int_0^{L/R} \sin n\beta x \sin m\beta x \, dx = \mu_{2s} \left(\frac{1}{2R}\right) \delta_{mn} \qquad \ldots \text{II - 2}$$

where δ_{mn} is the Kronecker delta.

The corresponding term for the "discrete" solution is

$$D(m,n) = \int_0^{L/R} \Big[\sum_{i=1}^{q-1} \mu_{2di}\, \delta(x-x_i) \sin n\beta x \sin m\beta x \Big] dx$$

$$= \sum_{i=1}^{q-1} \mu_{2di} \sin n\beta x_i \sin m\beta x_i \qquad \ldots \text{II} - 3$$

Now for equal rings $\mu_{2di} = \mu_{2d}$ and therefore

$$D(m,m) = \mu_{2d} \sum_{i=1}^{q-1} \sin \frac{m\pi i}{q} \sin \frac{m\pi i}{q} = \frac{\mu_{2d}q}{2} \qquad \ldots \text{II} - 4$$

Hence, after substitution from Eq. (II-1)

$$D(m,m) = \frac{\mu_{2s}L}{2R} = S(m,m) \qquad \ldots \text{II} - 5$$

This means that the diagonal terms in the "discrete" stability determinant are equal to those of the "smeared" case. On account of the Kronecker delta in Eq. (II-2) one can conclude that the first approximation of the "discrete" case is equal to the "smeared" solution for any number of rings and any n.

APPENDIX III

CONVERGENCE OF THE SOLUTION

In the "smeared" case the solution is in a closed form, whereas in the "discrete" case there is a series which converges to the solution.

For the interesting case, when there is a considerable difference between the "smeared" and discrete solutions, the convergence is slow. This is to be expected since rapid convergence means that the shape of the deflection is close to a simple sine curve, or to a combination of the first harmonics. When the effect of discreteness is appreciable the deflection shape is usually different from a simple sine curve.

Fortunately, however, the series for one ring, though it converges slowly, can be extrapolated easily. As can be seen from Fig. 8 , a semi-logaritmic plot of the difference between successive approximations approaches a straight line asymptotically. This means that the difference can be represented by a geometric series. Thus if we know three approximations, for example λ_i, λ_{i+1}, λ_i+2, we can estimate the limit

$$\lambda_\infty = \lambda_i - \frac{(\lambda_i - \lambda_{i+1})}{\dfrac{\lambda_{i+1}-\lambda_{i+2}}{\lambda_i - \lambda_{i+1}}}$$

When the shell has more than one ring, this is not true anymore and in the numerical work the 10^{th} approximation was then usually considered adequate. This rather arbitrary decision was based on the results obtained for one ring where the difference between λ_{10} and λ_∞ was usually less than 2-3%, except for large L/R values for which the difference was up to 5%.

TABLE I.

LOCAL BUCKLING OF TYPICAL RING STIFFENED SHELLS

$A_2/ah = 0.5$ $I_{22}/ah^3 = 5$ $e_2 = 5$

R/h	L/R	λ smeared	local buckling for one ring *	Number of rings required for general instability to predominate
50	0.5	644.9	428.9	2
	1.0	644.9	173.2	5
	1.5	644.9	108.7	7
100	0.5	1303	533.8	3
	1.0	1288	229.9	7
	2.0	1281	107.4	14
250	0.5	3228	755.7	5
	1.0	3224	345.9	10
500	0.5	6398	1013	8
	1.0	6398	477.2	16
	2.0	4660	231.7	26
1000	0.5	12860	1381	11
	1.0	12180	661.1	22
	2.0	6881	323.8	29
2000	0.5	25590	1903	16
	1.0	18520	924.0	26
	2.0	9957	454.9	33

* Torsional stiffness of rings is neglected.

TABLE 2

INFLUENCE OF SHELL GEOMETRY ON DISCRETENESS EFFECT

$A_2/ah = 0.1$ $I_{22}/ah^3 = 0.01$ $e_2/h = 1$

R/h	L/R	Z	λ smeared	λ discrete*	†	Δλ %
100	0.25	5.96	618.97	617.28	11	0.27
	0.5	23.85	324.44	315.76	10	2.68
	0.75	53.66	223.64	216.48	8	3.17
	1.0	95.39	172.76	163.24	7	4.92
	1.25	149.0	140.08	129.73	7	7.40
	1.5	214.6	117.20	108.80	6	7.16
	1.75	292.1	101.68	92.03	6	9.50
	2.0	381.6	91.90	81.10	6	11.75
	2.25	482 [illegible]	79.51	71.25	5	10.4
	2.50	596.3	71.78	62.52	5	12.9
	2.75	721.4	66.75	56.82	5	14.5
	3.0	858.5	63.35	52.75	5	16.7
500	0.25	29.8	1457.7	1417.5	21	2.76
	0.5	119.2	771.4	722.4	16	6.35
	0.75	268.3	525.3	481.7	13	8.30
	1.0	477.0	402.1	353.9	13	12.0
	1.25	745.2	323.4	278.1	11	13.9
	1.5	1073	272.5	228.3	10	16.2
	1.75	1461	231.8	189.8	9	18.2
	2.0	1908	204.4	163.4	9	20.0
	2.25	2415	181.3	141.2	8	22.1
	2.75	3607	148.6	114.7	7	22.8
	3.0	4293	136.8	102.5	7	25.1

TABLE 2. (CONT'D)

INFLUENCE OF SHELL GEOMETRY ON DISCRETENESS EFFECT

$A_2/ah = 0.1$ $\qquad I_{22}/ah^3 = 0.01$ $\qquad e_2/h = 1$

R/h	L/R	Z	λ smeared	λ discrete*	t	Δλ%
1000	0.25	59.6	2125.2	2036.6	26	4.17
	0.5	238.5				
	0.75	536.6	753.60	664.6	16	11.8
	1.0	953.9				
	1.25	1490	461.48	373.7	13	19.05
	1.5	2146	385.87	303.8	12	21.3
	1.75	2921	332.16	261.5	11	21.2
	2.0	3816	288.95	218.8	10	24.3
	2.25	4829	259.24	193.3	10	25.4
	2.5	5963	231.90	170.0	9	26.5
	2.75	7214	214.73	155.6	9	27.5
	3.0	8585	193.66	144.6	8	25.3

* Extrapolation to infinite order approximation.

TABLE 3

INFLUENCE OF RING AREA ON DISCRETENESS EFFECT

$L/R = 3$ $R/h = 250$

e_2/h	A_2/ah	I_{22}/ah^3	t	λ discrete*	λ smeared	Δλ%
1	0.01	0.001	6	58.33	58.62	0.5
	0.03	0.003	6	64.20	67.68	5.1
	0.05	0.005	6	68.10	76.45	10.9
	0.06	0.006	6	69.60	80.72	13.7
	0.08	0.008	6	72.70	89.08	18.5
	0.10	0.010	6	76.00	97.17	21.8
	0.12	0.012	6	78.30	105.02	26.4
-1	0.01	0.001	6	57.17	57.79	0.7
	0.03	0.003	6	61.20	65.25	6.2
	0.05	0.005	6	64.05	72.47	11.6
	0.06	0.006	6	65.30	76.00	14.0
	0.08	0.008	6	67.25	82.89	18.9
	0.10	0.010	6	68.90	89.56	23.1
	0.12	0.012	6	70.20	96.05	26.8
	0.14	0.014	6	71.40	102.34	30.1

* Extrapolation to infinite approximation

TABLE 4

EFFECT OF RING PARAMETERS - ECCENTRICITY OF RINGS.

L/R = 1 R/h = 500

e_2/h	A_2/ah	I_{22}/ah^3	λ smeared	λ discrete*	t	Δλ%
-1	0.01	0.001	252.45	251.56	12	0.35
	0.03	0.003	279.11	269.15	12	3.55
	0.05	0.005	307.71	284.50	12	7.50
	0.07	0.007	335.40	296.60	12	10.7
	0.085	0.0085	355.60	303.80	12	14.5
	0.10	0.010	373.70	310.10	12	17.0
	0.12	0.012	394.50	317.40	12	19.5
	0.14	0.014	414.70	324.20	12	21.5
	0.16	0.016	434.20	328.80	12	24.0
	0.18	0.018	453.30	334.00	12	26.2
-2	0.005	0.0005	263.96	261.60	12	0.89
	0.010	0.001	293.30	284.90	12	2.86
	0.015	0.0015	322.31	304.78	12	5.45
	0.020	0.002	351.10	322.10	12	8.29
	0.025	0.0025	378.76	337.27	12	10.9
	0.030	0.003	402.01	350.45	12	12.8
	0.035	0.0035	425.04	362.40	12	14.75
	0.040	0.004	447.87	373.00	12	16.7
	0.045	0.0045	470.50	381.90	12	18.8
-3	0.001	0.0001	247.63	247.25	13	0.15
	0.003	0.0003	274.87	271.60	12	1.05
	0.006	0.0006	314.32	303.70	12	3.40
	0.010	0.0010	367.12	340.05	12	7.36
	0.012	0.0012	390.74	355.85	12	8.94

TABLE 4 (Cont'd)

EFFECT OF RING PARAMETERS - ECCENTRICITY OF RINGS

L/R = 1 R/h = 500

e_2/h	A_2/ah	I_{22}/ah^3	λ smeared	λ discrete*	t	$\Delta\lambda$%
-3	0.015	0.0015	423.14	377.55	12	10.80
	0.018	0.0018	455.36	396.40	12	12.95
-4	0.001	0.0001	258.25	257.42	12	0.32
	0.003	0.0003	305.86	298.76	12	2.32
	0.005	0.0005	353.29	334.88	12	5.20
	0.007	0.0007	397.01	366.37	12	7.97
	0.009	0.0009	436.02	394.08	12	9.60
-10	0.0002	0.00002	264.52	263.65	12	0.33
	0.0004	0.00004	294.65	291.21	12	1.16
	0.0006	0.00006	324.77	317.18	12	2,32
	0.0008	0.00008	354.87	341.63	12	3.63
	0.0010	0.00010	384.66	364.42	12	5.27
	0.0012	0.00012	409.70	385.60	12	5.65
	0.0014	0.00014	434.74	407.20	12	6.35
	0.0016	0.00016	459.76	426.60	12	7.20

* Extrapolation to infinite approximation.

TABLE 5

INFLUENCE OF THE NUMBER OF RINGS ON DISCRETENESS EFFECT FOR DIFFERENT L/R AND R/h

$e_2/h = -1$ $A_2/ah = 0.15$ $I_{22}/ah^3 = 0.015$

R/h	L/R	Z	Number of Rings	t	λ smeared	λ discrete*	Δλ%
500	0.5	119.2	1	16	842.92	692.22	17.9
			2	16		743.38	11.8
			3	16		781.16	7.34
			4	16		806.99	4.27
			5	15		821.45	2.55
			6	15		828.69	1.69
			7	15		832.97	1.18
			8	15		835.64	0.86
			9	15		837.38	0.66
			10	15		838.56	0.52
			11	15		839.41	0.42
			12	15		840.04	0.34
			13	15		840.56	0.30
			14	15		840.86	0.25
			15	15		841.15	0.21
500	1	477.0	1	12	424.5	330.8	22.1
			2	12		351.6	17.2
			3	12		367.6	13.4
			4	11		380.5	10.4
			5	11		389.2	8.34
			6	11		396.9	6.52
			7	11		403.2	5.03
			8	11		408.3	3.81
			9	11		412.1	2.93
			10	11		414.9	2.27
			11	11		417.0	1.77

TABLE 5 (Cont'd)

INFLUENCE OF THE NUMBER OF RINGS ON DISCTETENESS EFFECT FOR DIFFERENT L/R AND R/h

$e_2/h = -1$ $A_2/ah = 0.15$ $I_{22}/ah^3 = 0.015$

R/h	L/R	Z	Number of Rings	t	λ smeared	λ discrete*	Δλ%
500	1	477.0	12	11	424.5	418.52	1.42
			13	11		419.67	1.14
			14	11		420.53	0.95
			15	11		421.20	0.78
500	2	1908	1	9	213.17	165.73	22.2
			2	8		170.11	20.2
			3	8		174.37	18.2
			4	8		178.51	16.1
			5	8		182.37	14.4
			6	8		185.71	12.9
			7	8		188.61	11.5
			8	8		191.18	10.3
			9	8		193.50	9.23
			10	8		195.66	8.20
			11	8		197.67	7.28
			12	8		199.56	6.38
			13	8		201.29	5.57
			14	8		202.85	4.85
			15	8		204.25	4.18
100	1	95.39	1	8	188.41	156.80	15.85
			2	7		171.72	9.10
			3	7		178.17	5.45
			4	7		182.38	3.20
			5	7		184.67	1.99

TABLE 5 (Cont'd)

INFLUENCE OF THE NUMBER OF RINGS ON DISCRETENESS EFFECT FOR DIFFERENT L/R AND R/h

$e_2/h = -1$ $A_2/ah = 0.15$ $I_{22}/ah^3 = 0.015$

R/h	L/R	Z	Number of Rings	t	λ smeared	λ discrete*	Δλ%
100	1	95.39	6	7	188.41	185.93	1.31
			7	7		186.67	0.92
			8	7		187.12	0.68
			9	7		187.42	0.52
			10				
			11	7		187.77	0.33
			12	7		187.88	0.28
			13	7		187.96	0.24
			14	7		188.03	0.20
2000	1	1908	1	17	852.69	652.78	23.5
			2	17		670.08	21.4
			3	17		690.07	19.1
			4	17		709.45	16.8
			5	17		727.51	14.7
			6	16		742.82	12.9
			7	16		754.46	11.5
			8	16		764.72	10.3
			9	16		774.01	9.20
			10	16		782.62	8.23
			11	16		790.70	7.28
			12	16		798.73	6.42
			13	16		805.15	5.49
			14	16		811.42	4.84
			15	16		816.98	4.18

* Tenth order approximation

TABLE 6

INFLUENCE OF NUMBER OF RINGS ON THE DISCRETENESS EFFECT BETWEEN CUT-OFF LINES

FOR RINGS OF VARYING STRENGTH.

$L/R = 1$ $R/h = 500$ $e_2/h = -2$

Number of Rings	A_2/ah	I_{22}/ah	t	λ smeared	λ discrete*	Δλ%
1	0.003	0.00225	12	255.13	254.22	0.36
	0.018	0.01350	12	357.4	331.1	7.36
	0.039	0.02925	12	475.5	400.5	15.7
2	0.042	0.03375	11	491.6	435.7	11.4
	0.060	0.04500	11	586.7	486.1	17.1
	0.096	0.07200	11	736.1	563.0	23.5
3	0.099	0.07425	11	748.0	611.1	18.7
	0.120	0.09000	11	830.2	655.1	21.1
	0.168	0.12600	10	1007	733.7	27.1
4	0.171	0.12825	10	1016	784.4	22.8
	0.210	0.15750	10	1123	843.0	25.0
	0.280	0.21000	10	1303	930.2	28.6
5	0.288	0.21600	10	1323	1010	23.6
	0.360	0.27000	10	1492	1097	26.4
	0.416	0.31200	10	1615	1155	28.5
6	0.432	0.32400	9	1650	1252	24.1
	0.512	0.38400	9	1806	1326	26.8
	0.608	0.45600	9	1943	1406	27.6
7	0.624	0.46800	9	1965	1525	22.4
	0.800	0.60000	9	2189	1671	23.7
	0.896	0.67200	9	2300	1741	24.3
8	0.912	0.68400	9	2318	1889	18.5

* tenth order approximation

TABLE 7

COMPARISON BETWEEN DISCRETE AND "SMEARED" NON-UNIFORM 3-RING STIFFENED CYLINDERS UNDER LATERAL PRESSURE*

$R/h = 500$ $L/R = 1$ $e_2/h = 2$

A_2/ah	I_{22}/ah^3	λ smeared	λ discrete	smeared non-uniform**	discrete non-uniform	smeared $\Delta\lambda\%$	discrete $\Delta\lambda\%$
0.08	0.06	758.1	727.7	967.1	767.4	27.6	5.41
0.10	0.0667	854.7	809.8	1109	858.1	29.8	5.96
0.12	0.08	948.4	896.2	1246	943.9	31.4	5.85

* The non-uniformity is a sinusoidal height variation of rings of rectangular cross-section.

** Half the cross-sectional area of the rings is constant and half varies sinusoidally.

TABLE 8

DISCRETENESS EFFECT IN RING-STIFFENED CYLINDRICAL SHELLS UNDER AXIAL COMPRESSION

Shell No.	R/h	L/R	e_2/h	A_2/ah	I_{22}/ah^3	Ring Spacing a/h	t discrete	No. of rings	λ smeared	λ discrete*	Δλ%
1	395	2.025	3.00	0.878	1.83	57.1	10	13	5515	5416	1.79
2	384	2.030	2.93	0.900	1.70	55.7	10	13	5354	5267	1.63
3	415	2.030	3.16	0.834	2.13	60.2	10	13	5805	5693	1.92
4	373	0.526	-0.877	0.347	0.0169	10.9	0	17	5689	5689	0.00
5	314	0.465	-0.690	0.175	0.00209	9.1	0	15	4453	4453	0.00
6	404	0.750	-0.936	0.447	0.00333	11.7	0	25	6387	6387	0.02
7	384	0.758	0.914	0.410	0.0227	10.8	15	26	5067	5067	0.00
8	667	0.515	1.39	0.878	0.2285	19.1	17	17	9044	9043	0.02

* eigth order approximation.

TABLE 9

CONVERGENCE OF THE APPROXIMATIONS FOR "DISCRETE" BUCKLING LOADS

$A_2/ah = 0.1$ $e_2/h = 1.$ $I_{22}/ah = 0.01$

L/R	R/h	No.	n	λ	Δλ
250	3	1	1	97.173	
					5.079
		2	1-3	92.094	
					2.413
		3	1-5	89.681	
					1.835
		4	1-7	87.856	
					1.527
		5	1-9	86.329	
					1.321
		6	1-11	85.008	
					1.158
		7	1-13	83.850	
					1.018
		8	1-15	82.832	
					0.889
		9	1-17	81.943	
					0.776
		10	1-19	81.167	
100	0.5	1	1	324.436	
					4.599
		2	1-3	319.837	
					1.493
		3	1-5	318.345	
					0.771
		4	1-7	317.574	
					0.471
		5	1-9	317.103	
					0.316
		6	1-11	316.757	
					0.226
		7	1-13	316.561	
					0.170
		8	1-15	316.391	
					0.133
		9	1-17	316.258	
					0.106
		10	1-19	316.152	

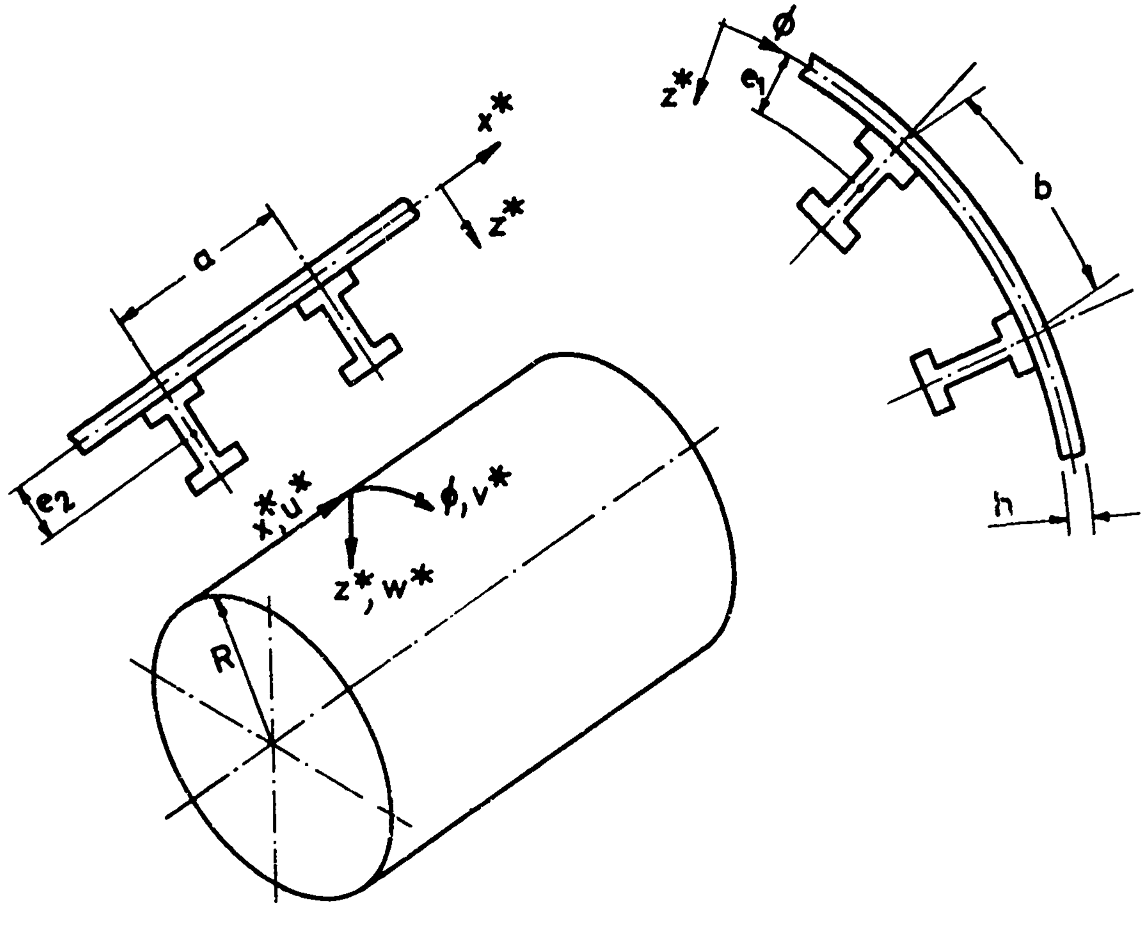


FIG. 1 NOTATION

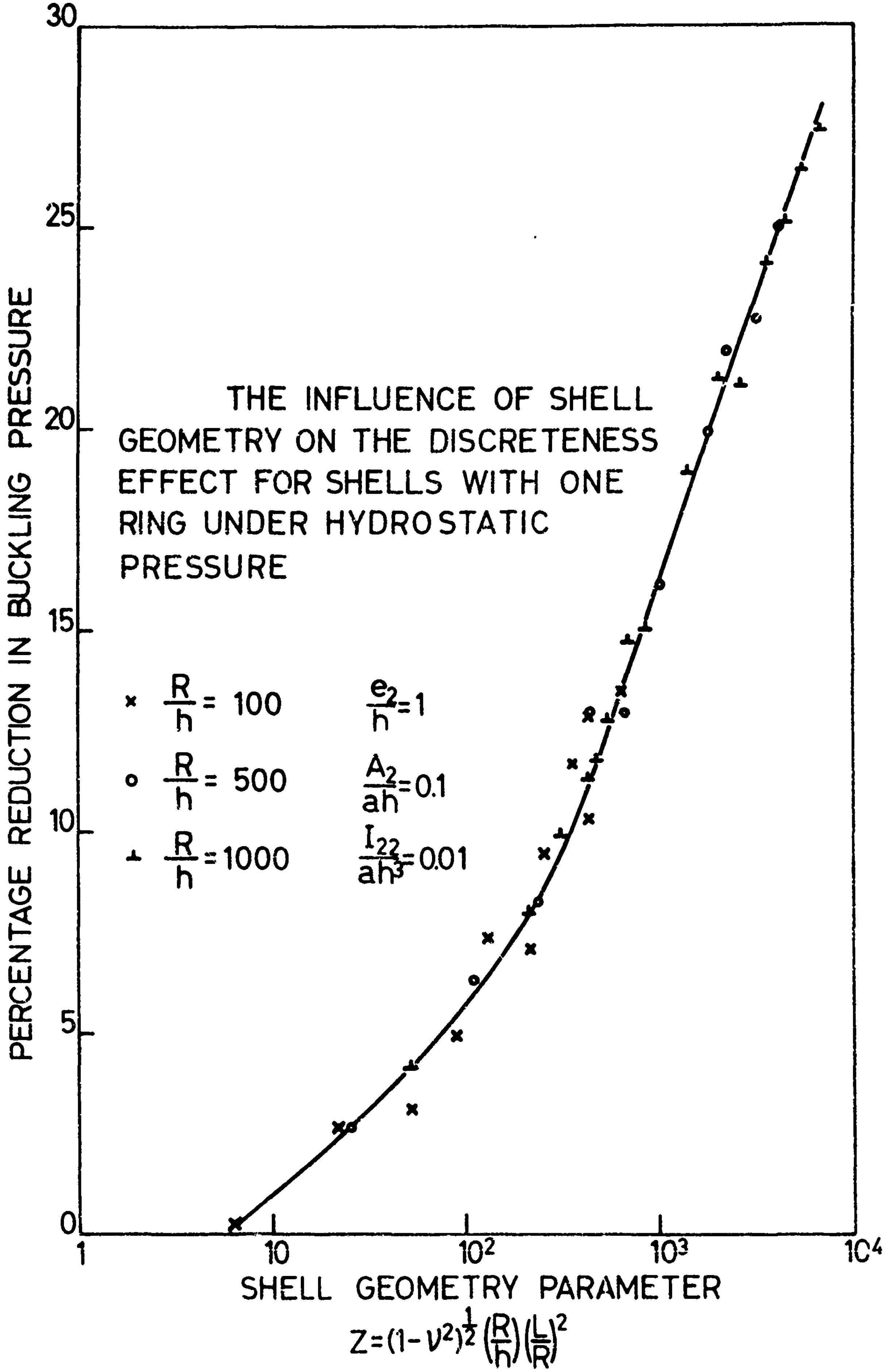

THE INFLUENCE OF SHELL GEOMETRY ON THE DISCRETENESS EFFECT FOR SHELLS WITH ONE RING UNDER HYDROSTATIC PRESSURE
PERCENTAGE REDUCTION IN BUCKLING PRESSURE
SHELL GEOMETRY PARAMETER
$Z=(1-\nu^2)^{\frac{1}{2}}\left(\frac{R}{h}\right)\left(\frac{L}{R}\right)^2$
× $\frac{R}{h}$ = 100
o $\frac{R}{h}$ = 500
+ $\frac{R}{h}$ = 1000
$\frac{e_2}{h}$ = 1
$\frac{A_2}{ah}$ = 0.1
$\frac{I_{22}}{ah^3}$ = 0.01
0
5
10
15
20
25
30
1
10
10^2
10^3
10^4

FIG. 2

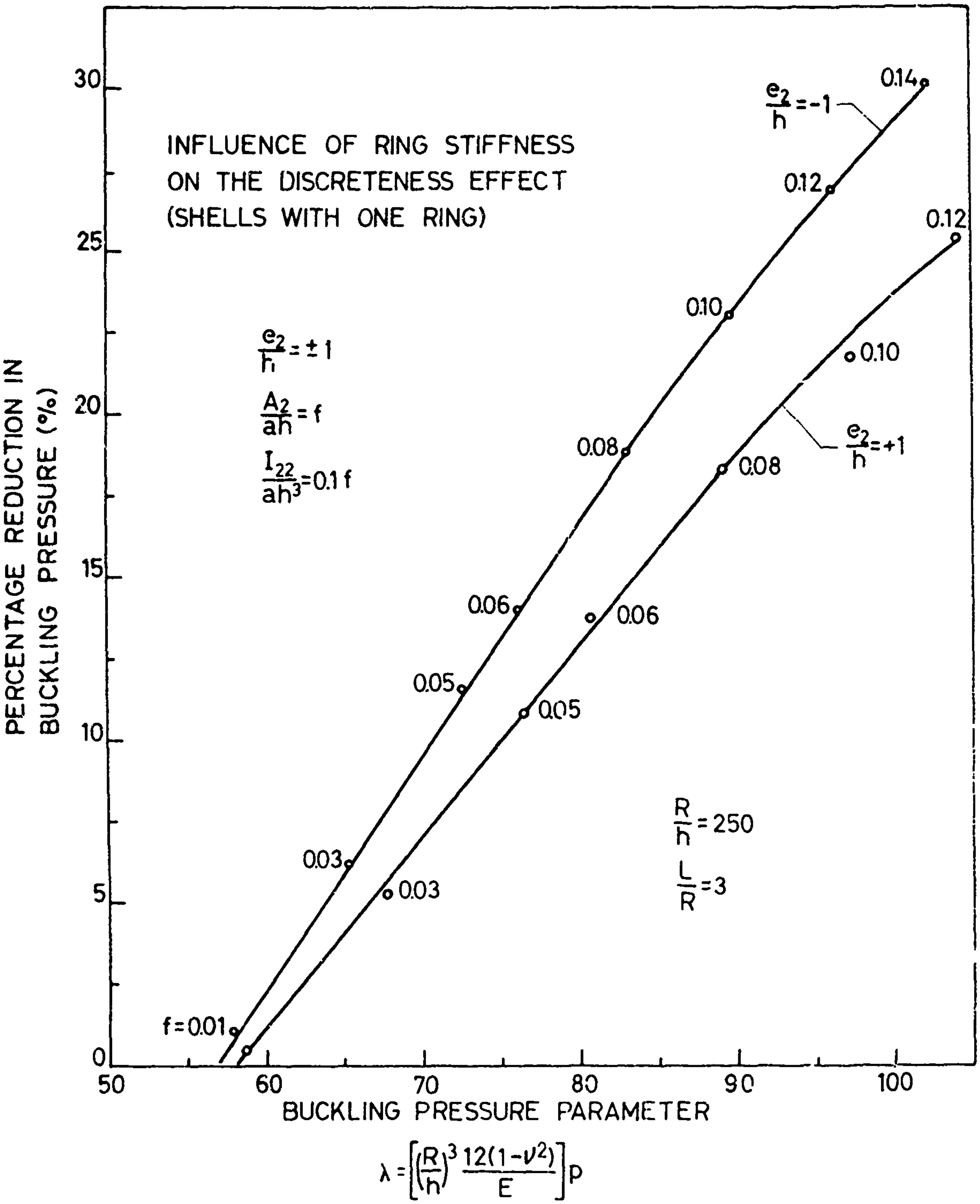
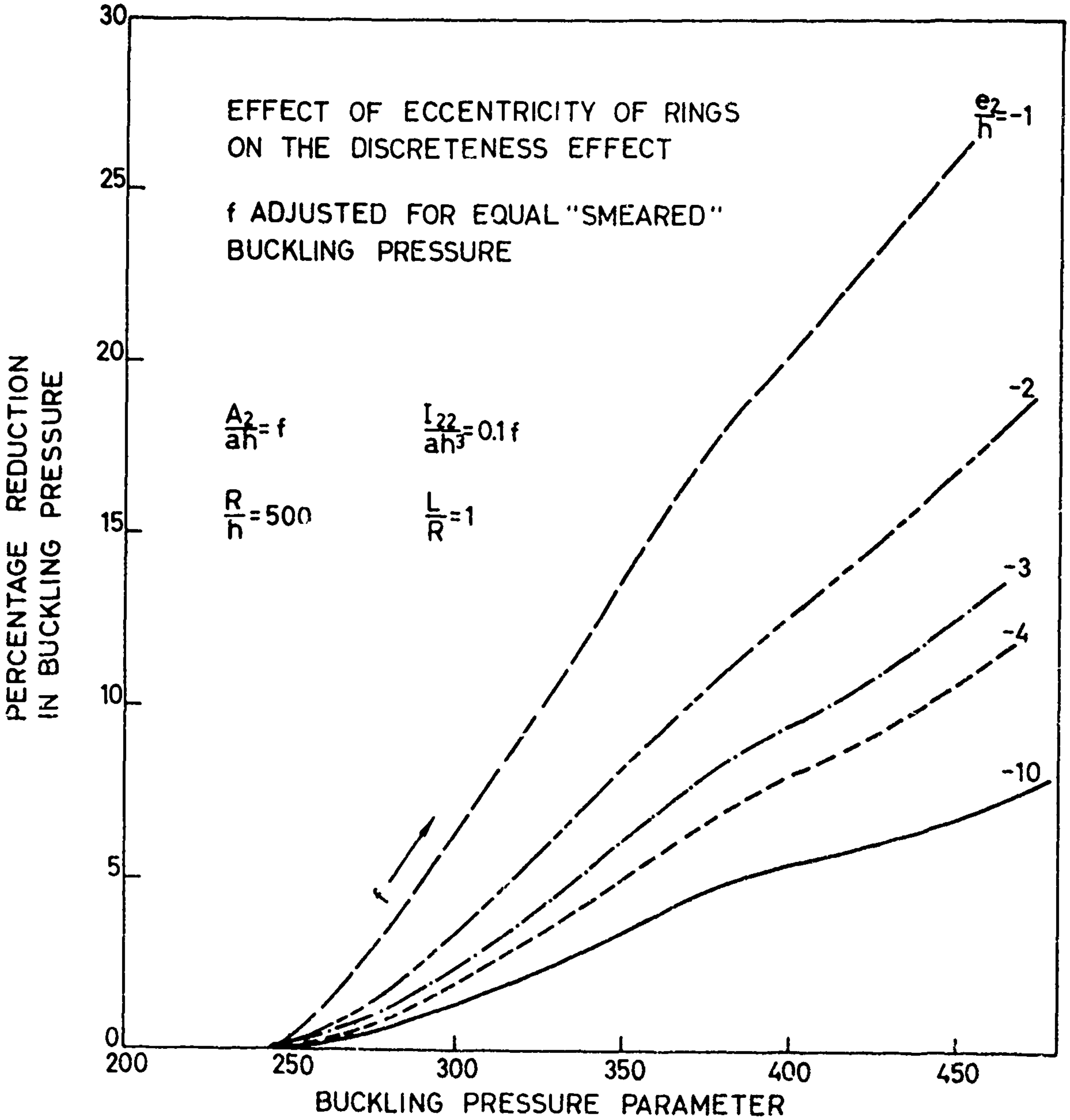
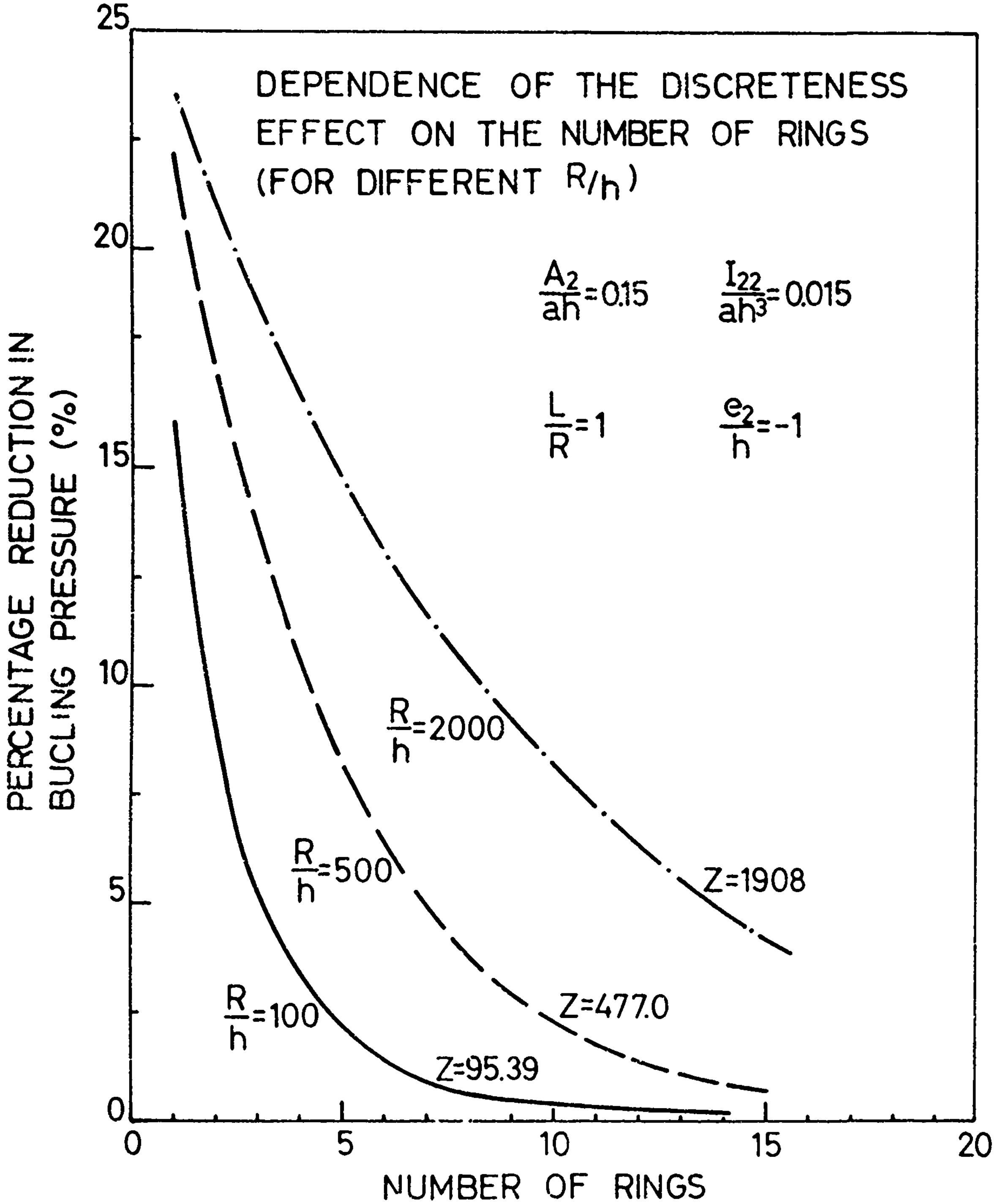

INFLUENCE OF RING STIFFNESS
ON THE DISCRETENESS EFFECT
(SHELLS WITH ONE RING)
$\frac{e_2}{h} = \pm 1$
$\frac{A_2}{ah} = f$
$\frac{I_{22}}{ah^3} = 0.1 f$
$\frac{e_2}{h} = -1$
$\frac{e_2}{h} = +1$
$\frac{R}{h} = 250$
$\frac{L}{R} = 3$
f = 0.01
0.03
0.05
0.06
0.08
0.10
0.12
0.14
PERCENTAGE REDUCTION IN BUCKLING PRESSURE (%)
BUCKLING PRESSURE PARAMETER
$\lambda = \left[\left(\frac{R}{h}\right)^3 \frac{12(1-\nu^2)}{E}\right] p$
0
5
10
15
20
25
30
50
60
70
80
90
100

FIG 3



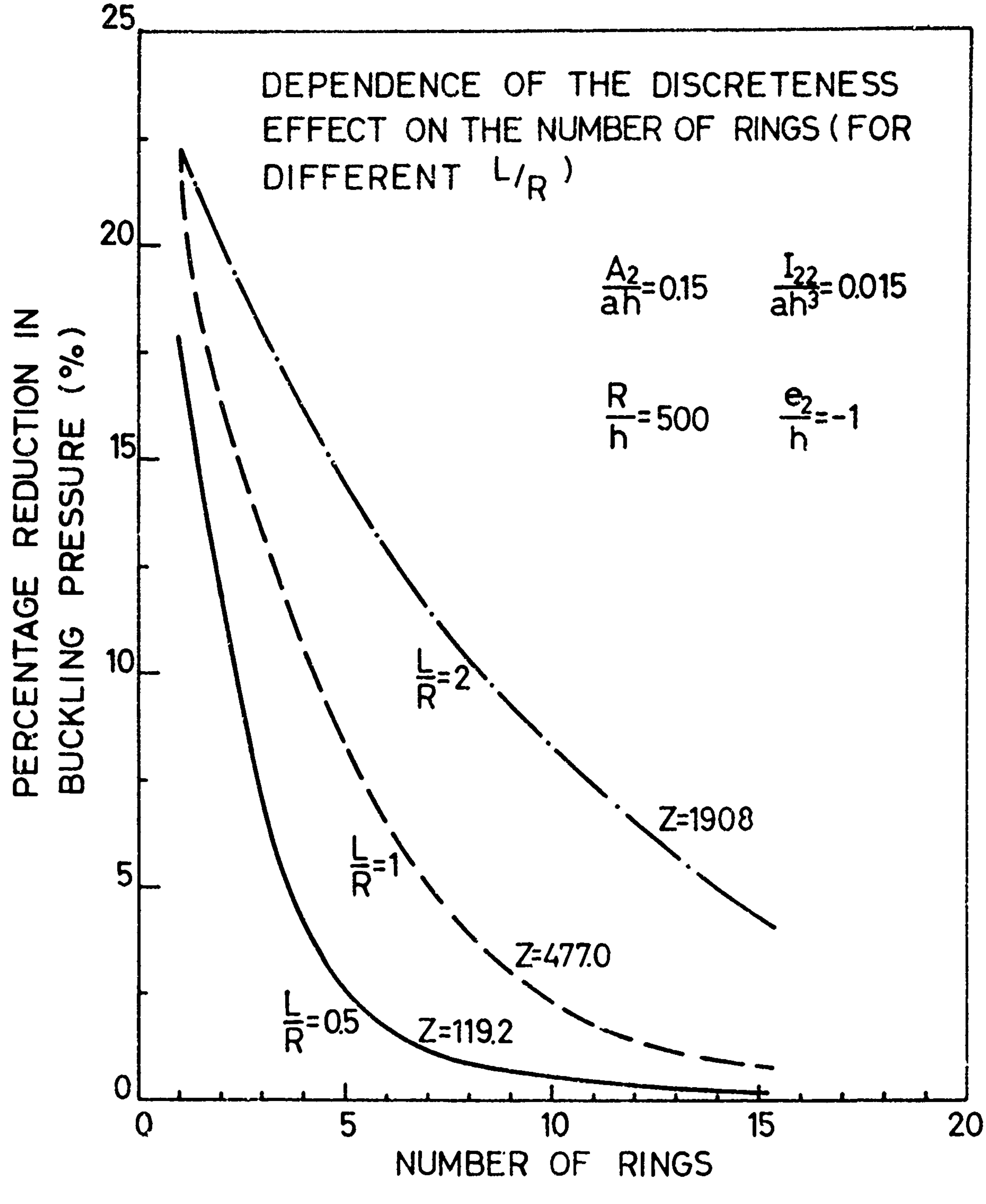
$$\lambda = \left[\left(\frac{R}{h}\right)^3 \frac{12(1-\nu^2)}{E}\right] p$$

FIG 4



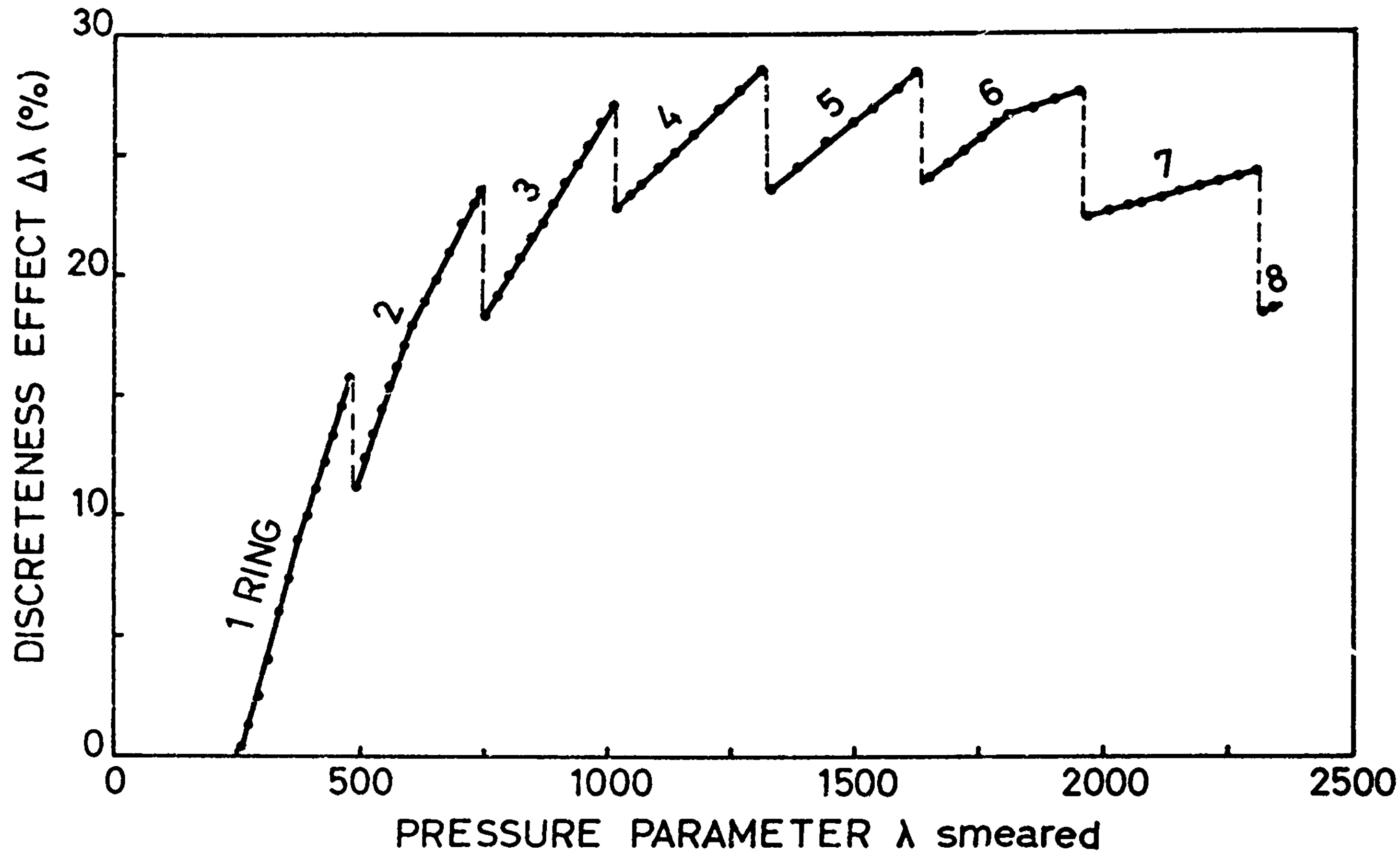
DEPENDENCE OF THE DISCRETENESS EFFECT ON THE NUMBER OF RINGS (FOR DIFFERENT R/h)
$\frac{A_2}{ah}=0.15$
$\frac{I_{22}}{ah^3}=0.015$
$\frac{L}{R}=1$
$\frac{e_2}{h}=-1$
$\frac{R}{h}=2000$
$\frac{R}{h}=500$
$\frac{R}{h}=100$
Z=1908
Z=477.0
Z=95.39
PERCENTAGE REDUCTION IN BUCLING PRESSURE (%)
NUMBER OF RINGS
0
5
10
15
20
25

FIG. 5



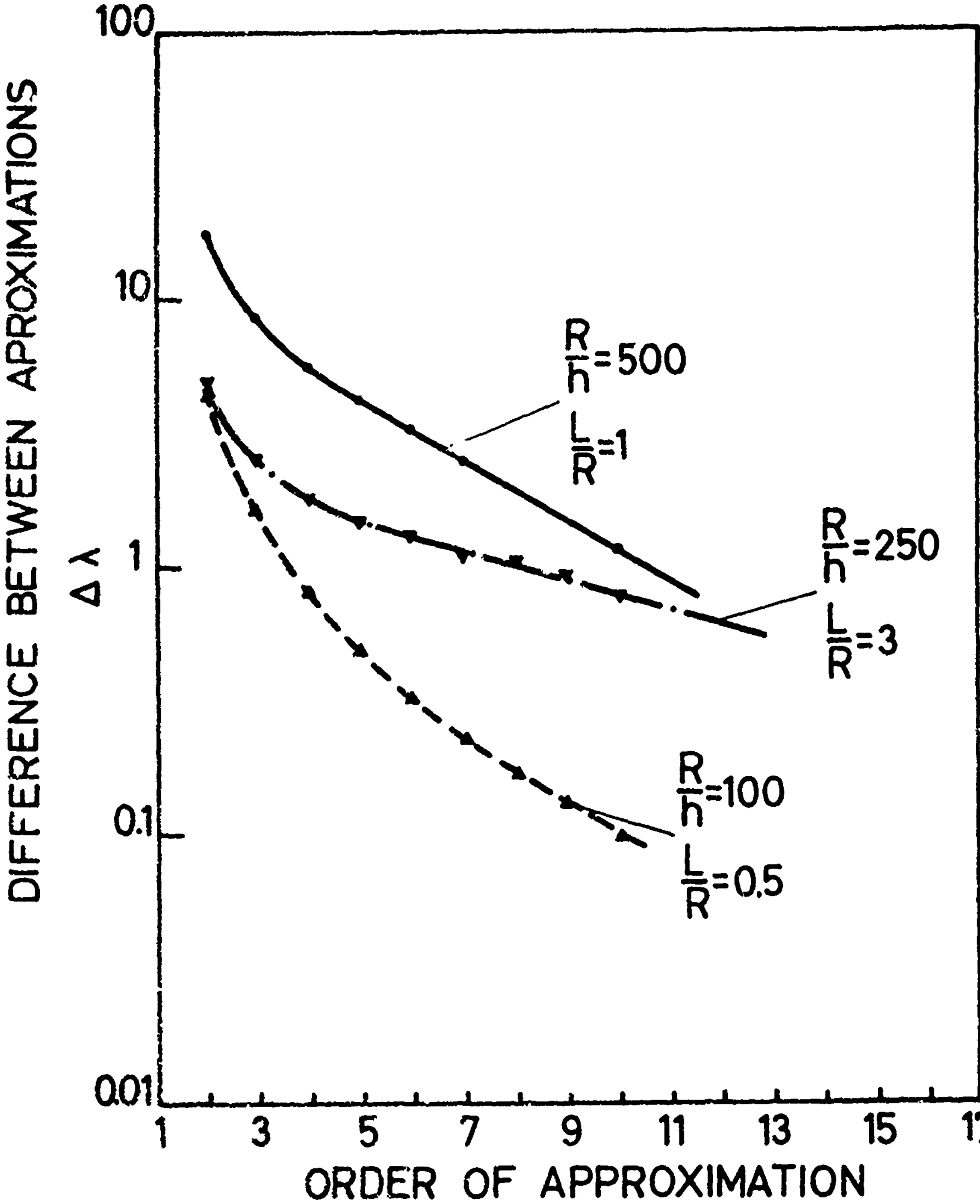
DEPENDENCE OF THE DISCRETENESS EFFECT ON THE NUMBER OF RINGS (FOR DIFFERENT L/R)
$\frac{A_2}{ah}=0.15$
$\frac{I_{22}}{ah^3}=0.015$
$\frac{R}{h}=500$
$\frac{e_2}{h}=-1$
$\frac{L}{R}=2$
Z=1908
$\frac{L}{R}=1$
Z=477.0
$\frac{L}{R}=0.5$
Z=119.2
PERCENTAGE REDUCTION IN BUCKLING PRESSURE (%)
NUMBER OF RINGS
0
5
10
15
20
25

FIG. 6



INFLUENCE OF THE NUMBER OF RINGS ON THE DISCRETENESS EFFECT FOR RINGS OF VARYING STRENGTH

FIG. 7



CONVERGENCE OF APPROXIMATIONS

FIG. 8

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) TECHNION RESEARCH AND DEVELOPMENT FOUNDATION, HAIFA, ISRAEL.	2a. REPORT SECURITY CLASSIFICATION 2b GROUP

3 REPORT TITLE

BUCKLING OF DISCRETE RING STIFFENED CYLINDRICAL SHELLS

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

SCIENTIFIC REPORT No. 2.

5 AUTHOR(S) (Last name, first name, initial)

Singer, Josef and Haftka, Raphael.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
AUGUST 1967	43 + 8 Figures	24

8a. CONTRACT OR GRANT NO. AF 61(052)-905 b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) TAE REPORT No. 67. 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY AIR FORCE OFFICE OF SCIENTIFIC RESEARCH OAR, European Office of Aerospace Research

13 ABSTRACT

The buckling of ring-stiffened cylinders is studied by a "discrete" approach, in which the rings are considered as linear discontinuities represented by the Dirac delta function. The analysis is a linear Donnell type theory that takes account of the eccentricity of stiffeners. Buckling loads under hydrostatic pressure, lateral pressure and axial compression are compared with those obtained by "smeared-stiffener" theory for an extensive range of geometries. The discreteness effect depends very strongly on the geometry of the shell and the eccentricity of the rings. Significant discreteness effects are found for hydrostatic pressure loading.

DD FORM 1 JAN 64 1473

Unclassified
Security Classification

Unclassified

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Buckling Stiffened Cylindrical Shells						
2. Discrete Stiffeners						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY, LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ____________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ____________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ____________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

Unclassified

Security Classification